Manager-Investor game: evolve of honesty and incredulity

By Haogong Tong*

In this article, I designed a game called Manager-Investor game. The manager and investor chip in same amount of money in a project which can yield high profit or low profit. Players are supposed to split the profit but only the manager knows the real profit and investor can only get the information from manager. The game model their behaviors as honest/dishonest manager and credulous/incredulous investor. I found that if the cost of revealing a lie is sufficiently small, the society does not have evolutionarily stable strategies. But by letting managers cover the investigation cost, society can evolve into a status where managers never lie and investors never trust. *Keywords: Manager-Investor game, Evolutionary game theory, Evolutionarily stable strategy*

I. Introduction

This article is an extension of my thoughts occurred when I was on vacation in Las Vegas with my friend Henry. One night, I was eager to play in the poker tournament at Harrah's Casino. The entry fee was 60 dollars but cash only. I only had 30 dollars in my wallet, so Henry and I made an agreement: he would chip in another 30 dollars, and if I won any money in the tournament, we would split the profit. The first prize was 500 dollars and the second prize was only 300 dollars. While I was playing, Henry went to a bar for drinks, so he had no idea how the game was going on or how much money I won. Instead, when we met back at the hotel, I gave him his share. I have always been honest with my friends, but for the purpose of this article, assume I could be dishonest. If I won the first prize, I could tell Henry that I won the second prize to keep 350 dollars to myself. Henry may not believe me if I said I only won the second prize since he knows I am an excellent poker player. Interestingly, this became a game between Henry and me. I could choose to lie or not, and Henry can opt to believe me or not. If I won the first prize, I would have the incentive to tell him that I only won the second prize. And if he believed me, I could retain more revenue to myself. If he disbelieved me, he would have to spend time or even money to prove that I lied (He might go back to the casino and asked the manager to pull out a list of the prize winners), and I would have to spend my time or even money in the investigation (I would be asked to go back to the casino with him). What really happened that night was that I lost all of our money in the tournament, but

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this awkward situation inspired an entertaining game. I formalized this game in Section II as *Manager-Investor game*.

This game has some practical significance. The agreement between Henry and me is practically a mutual fund in nature. People put money in mutual funds, and the fund manager is responsible for investing their money and returning the profits. We observe the fund's profit through financial statements, which could be manipulated by the fund manager. About half of the fund managers in the US invest their own money in their funds. Investing in own funds can encourage fund managers to invest more wisely, but as analyzed above, since investors usually do not have complete information about how projects really went on, the manager might be motivated to lie about the annual profit. Without third party supervision, fund manager may under-report the annual revenue so he can retain more profit to himself. One may argue that fund managers are commonly considered to have incentive to over-report the revenue instead of under-report, which I believe is also true in some cases. For example, managers of closed-end funds [1] may over-report the revenue because closed-end funds are publicly traded and over-reporting revenues will increase the market demand and raise the price of shares. In such closed-end funds, the profits gained by investors depend not only on the investment performance but share price as well. But in this article, I only study the behavior where investment outcome is the only determinant of profit, such as in open-end funds, private equity funds and other investment agreements like the one between Henry and me. In these scenarios over-reporting revenue is not profitable. I focused solely on the incentive of under-reporting revenue caused by asymmetric information between the fund manager and his investors about the projects' real profit.

The classical game-theoretic approach, where two players directly play against each other and choose strategies, may not fully capture the nature of this Manager-Investor game because an investor is not necessarily limited to one specific manager. Moreover, investors may not be rational and may not have compete information about the probability distribution of the project. As explained in Section III, it is more appropriate to study investors and managers as a society. A standard approach studying such issue is to use evolutionary game theory, which was formally developed by Smith (1980), to analyze the dynamics of the proportion of honest managers and credulous investors. Evolutionary game theory differs from classical game theory by focusing more on the dynamics of strategy change as influenced not solely by the quality of the various competing strategies, but by the effect of the frequency with which those various competing strategies are found in the population (Easley and Kleinberg, 2010). The remainder of this article is as organized as follows: Section II established a formal model for Manager-Investor game with assumptions on rationality, Section III relaxed the assumption of ra-

[1] Defined by `http://www.investopedia.com/terms/c/closed-endinvestment.asp`, a closed-end fund is a publicly traded investment company that raises a fixed amount of capital through an initial public offering (IPO). The fund is then structured, listed and traded like a stock on a stock exchange.

tionality and extended the game to an evolutionary game, Section IV further discussed how a particular law could change the evolutionarily stable strategies of the game, and Section V summarized the conclusions.

II. The Manager-Investor game with rationality and complete understanding of game's structure

Let us describe the Manager-Investor game in a formal model. Suppose an investor (Henry) and a manager (me) are investing in a project (poker tournament), they both chip in the same amount of money and agree to split the profit equally. For simplicity, assume that the project can generate a high-profit H or a low-profit L ($H > L$) with equal chance. The manager can observe the profit generated from the project but the investor can't. Instead, the investor hears it from the manager. If the project made high profit, the manager can lie and tell the investor that the profit was low. Then, if the investor believed him, the manager would give out $\frac{L}{2}$ and keep the rest $H - \frac{L}{2}$ to himself. If the investor didn't believe the manager and committed some resource to reveal the truth, then they would split the real profit equally as agreed, but both players would suffer an *investigation cost* c ($c > 0$) because both of them lost time or money to the investigation.

Let I denote the investor and M denote the manager.

DEFINITION 1: *A player's action set Q is a set which contains all the actions that a player can do. The elements in an action set are called actions, denoted by q.*

A manager's action set Q_M is $\{Lie, Truth\}$ and an investor's action set Q_I is $\{Believe, Disbelieve\}$.

DEFINITION 2: *A manager's (pure) strategy is a pair of actions (q_M^H, q_M^L) indicating what he will do with high-profit H and low- profit L. An investor's (pure) strategy is a pair of actions (q_I^H, q_I^L) indicating how he will react if he is told the project receives high profit and is told low profit. A player's strategy set S is a set containing the strategies available to him.*

The game was summarized by the decision tree in Figure 1. Manager's payoff is written before investor's payoff at the terminal node. Note that a rational manager will not lie if the project received low profit and a rational investor will not challenge the manager if the manager told him the project received high profit. The manager's strategy set S_M contains two strategies: $(Lie, Truth)$ and $(Truth, Truth)$ and the investor's strategy set S_I also includes two strategies: $(Believe, Disbelieve)$ and $(Believe, Believe)$. The information set indicates that when an investor is told that the project received low profit, he can't say whether the project really received low profit or the manager is lying.

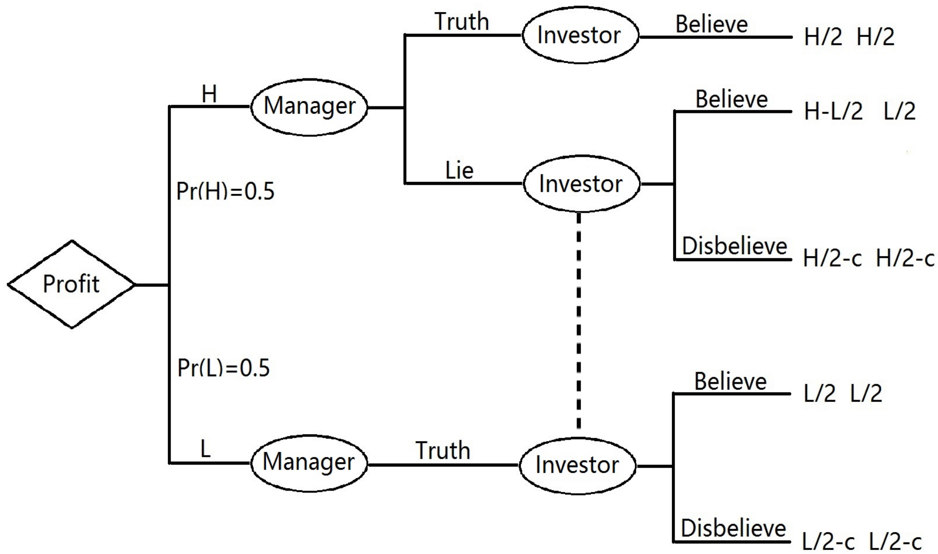


FIGURE 1. DECISION TREE OF MANAGER-INVESTOR GAME

DEFINITION 3: *A manager is a dishonest manager M_1 if he adopts the strategy $(Lie, Truth)$, otherwise he is an honest manager denoted by M_2. An investor is a credulous investor I_1 if he adopts the strategy $(Believe, Believe)$, otherwise he is an incredulous investor I_2.*

ASSUMPTION 1: *Both players have full knowledge of the structure of the game and both players are rational.*

ASSUMPTION 2: *Both players in Manager-Investor game are risk-neutral.*

Under Assumption 1, a player should try to work out his opponent's strategy and maximize his payoffs accordingly. Such effort leads to Nash equilibrium. Although the Manager-Investor game is a sequential game, due to the information set, players can decide their strategies simultaneously before the profit is solved. The simultaneous game described by Table 1 is equivalent to the sequential game in Figure 1. Both players will have an expected payoff because the profit can be high or low with equal probability. For example, if the manager plays strategy M_1 and the investor plays I_1, the manager's expected payoff is $0.5\times(H-\frac{L}{2})+0.5\times\frac{L}{2}=\frac{H}{2}$ and the investor's expected payoff is $0.5\times\frac{L}{2}+0.5\times\frac{L}{2}=\frac{L}{2}$. Similarly, Table 1 calculated all the expected payoffs in each scenario.

If $c>\frac{H-L}{4}$, the game has a pure strategy Nash Equilibrium shown as the investor is credulous and the manager is dishonest. If $c\leq\frac{H-L}{4}$, the Manager-Investor game has a mixed strategy Nash equilibrium. Let P_s denote the probability with which a player plays strategy s in a mixed strategy Nash equilibrium.

TABLE 1—MATRIX OF EXPECTED PAYOFF IN MANAGER-INVESTOR GAME

		Investor I_1	Investor I_2
Manager	M_1	$(\frac{H}{2}, \frac{L}{2})$	$(\frac{H+L}{4} - c, \frac{H+L}{4} - c)$
	M_2	$(\frac{H+L}{4}, \frac{H+L}{4})$	$(\frac{H+L}{4} - \frac{c}{2}, \frac{H+L}{4} - \frac{c}{2})$

A mixed strategy Nash equilibrium in Manager-Investor game could be expressed by $([M_1, P_{M_1}; M_2, P_{M_2}], [I_1, P_{I_1}; I_2, P_{I_2}])$. The mixed strategy Nash equilibrium is calculated and shown in Equation 1.

$$\begin{aligned} &NE_{Manager-Investor} = \\ &([M_1, \frac{2c}{H-L-2c}; M_2, \frac{H-L-4c}{H-L-2c}], [I_1, \frac{2c}{H-L+2c}; I_2, \frac{H-L}{H-L+2c}]) \end{aligned} \tag{1}$$

THEOREM 1: *Under Assumption 1 and 2, if investigation cost is sufficiently small ($c \leq \frac{H-L}{4}$), then a decrease in investigation cost will reduce the probability of lying and the probability of trust in the Nash equilibrium of Manager-Investor game.*

PROOF:
If $c \leq \frac{H-L}{4}$, we have:

$$\frac{dP_{M_1}}{dc} = \frac{2(H-L)}{(H-L-2c)^2} > 0 \qquad \frac{dP_{I_1}}{dc} = \frac{2(H-L)}{(H-L+2c)^2} > 0$$

∎

Theorem 1 shows that if players are rational and cognitive of the game's structure, one can reduce the investigation cost to achieve an honest status. From the perspective of a policy maker, the investigation cost is a dead weight loss. To improve efficiency, a policy maker should reduce the expected loss in investigation. Reducing c can raise the probability of investigation, so the overall effect of reducing c on deadweight loss is still uncertain. The expected deadweight loss is $E(DWL) = 2c \times P_{I_2} \times (P_{M_1} + P(profit = L) \times P_{M_2}) = 2c \times \frac{H-L}{H-L+2c} \times (\frac{2c}{H-L-2c} + 0.5 \times \frac{H-L-4c}{H-L-2c}) = \frac{c(H-L)^2}{(H-L)^2-4c^2}$, which is an increasing function of c. Therefore, reducing investigation cost can reduce the deadweight loss caused by investigation because the decrease in investigation cost won't be completely offset by the increase in the probability of investigation. This conclusion is summarized in Theorem 2.

THEOREM 2: *Under Assumption 1 and 2, if investigation is sufficiently small ($c < \frac{H-L}{4}$), then reducing investigation cost can reduce the expected deadweight loss in Manager-Investor game.*

Theorem 2 shows that if the investigation cost is sufficiently small, any further decrease in investigation cost can improve social efficiency.

III. The evolutionary approach of Manager-Investor game

In this section, I used evolutionary game theory to study the Manager-Investor game. From the classical game theoretic perspective, a player is expected to play by the Nash equilibrium strategies. A major presumption of Nash equilibrium is the cognitive abilities of the players. Players are assumed to be aware of the structure of the game and consciously try to predict the moves of their opponents and to maximize their own payoffs. From the perspective of evolutionary game theory, the players are not required to be rational at all, but only to have strategies that are passed on to their progeny. In short, the notion of player is displaced by that of strategy, and consequently the notion of a player's knowledge, complete or incomplete, is dispensed with. What drives systems is not the rationality of the players but the differential success of the strategies (Binmore, 2007). The distinction between classical approach and evolutionary approach is that from the classical perspective, players get to the Nash equilibrium through complete rationality, but in evolutionary approach, players get to the equilibrium by trial-and-error. I believe the evolutionary approach is more suitable for solving Manager-Investor game. I find it unrealistic to argue that investors are rationally maximizing their expected payoffs when deciding whether to be credulous because the investors usually do not have complete information about the probability distribution of profit or how much it costs to confront the manager. In this section, I do not assume the players have full knowledge of the game's structure. And as we can see later in this section, players improve their expected payoffs by trial-and-error instead of complete rationality.

Since we do not assume the investor understands the probability distribution of profits, an investor may also disbelieve the manager even when the manager gives him $\frac{H}{2}$. But it is certain that after trial-and-error, investors should realize that challenging the manager is not profitable if the manager returns $\frac{H}{2}$ to him. $(Disbelieve, Believe)$ is always inferior to $(Believe, Believe)$ and $(Disbelieve, Disbelieve)$ is always inferior to $(Believe, Disbelieve)$. So, we can remove the strategies in which $q_I^H = Disbelieve$ from investor's strategy set S_I.

Suppose there are two groups in a society: a group of managers and a group of investors. Assume that the population is sufficiently large, then we can represent the state of the population by simply keeping track of what proportion of managers follow the strategies M_1 and M_2, and what proportion of investors follow strategy I_1 and I_2. In the manager group, the proportion of dishonest managers is x $(0 \leq x \leq 1)$, and the proportion of credulous investors in the investor group is y $(0 \leq y \leq 1)$. Therefore, the society described in this game could be defined by a pair of proportions (x, y).

Managers and investors are repeatedly and randomly paired to play the game described in Figure 1. Players' types are defined by their (pure) strategies; Table

1 shows the expected payoffs. I do not assume players have full knowledge of Table 1.

Let $\Delta F(s_1, s_2)$ denote the change in utility for a player whose strategy is s_1 being paired with a player whose strategy is s_2, e.g., according to Table 1, $\Delta F(M_1, I_1) = \frac{H}{2}$ and similarly, $\Delta F(I_2, M_1) = \frac{H+L}{4} - c$. And let u_{M_i} and u_{I_j} denote the total utility of players following strategy M_i and I_j respectively; furthermore, suppose that each individual in the population has an initial utility of 0. The average utility of being an M_1 at time t could be expressed in terms of the population proportions and payoff values as $u_{M_1}(t) = y(t)\Delta F(M_1, I_1) + (1 - y(t))\Delta F(M_1, I_2)$. Similarly, we have the average utilities of all four types of players:

$$\begin{aligned} u_{M_1}(t) &= y(t)\Delta F(M_1, I_1) + (1 - y(t))\Delta F(M_1, I_2) \\ u_{M_2}(t) &= y(t)\Delta F(M_2, I_1) + (1 - y(t))\Delta F(M_2, I_2) \\ u_{I_1}(t) &= x(t)\Delta F(I_1, M_1) + (1 - x(t))\Delta F(I_1, M_2) \\ u_{I_2}(t) &= x(t)\Delta F(I_2, M_1) + (1 - x(t))\Delta F(I_2, M_2) \end{aligned} \tag{2}$$

Let $\bar{u}_M$ and $\bar{u}_I$ be the average utility of the entire managers' group and the whole investors' group at time t respectively.

$$\begin{aligned} \bar{u}_M(t) &= x(t)u_{M_1}(t) + (1 - x(t))u_{M_2}(t) \\ \bar{u}_I(t) &= y(t)u_{I_1}(t) + (1 - y(t))u_{I_2}(t) \end{aligned} \tag{3}$$

If we assume that the change in the strategy frequency from one generation to the next is small, these changes may be approximated by the differential equations offered by Taylor and Jonker (1978):

$$\frac{dx}{dt} = x(u_{M_1} - \bar{u}_M) \qquad \frac{dy}{dt} = y(u_{I_1} - \bar{u}_I) \tag{4}$$

Equation 4 is known as the *replicator dynamics*. These replicator equations in the context of evolutionary biology show the growth rate of the proportion of organisms using a certain strategy and that rate is equal to the difference between the average payoff of that strategy and the average payoff of the population as a whole (Samuelson, 2002). Equation 4 has two excellent properties. First, it captures the essence of social evolution. People tend to join the type whose utility is above social average. Second, since the dynamics of the proportion of M_2 and I_2 should have the same mathematical form as in Equation 4, it can be easily checked that Equation 4 guarantees that the proportion of M_1 and the proportion of M_2 always add up to 1, and proportion of I_1 and proportion of I_2 also always add up to 1.

The society reaches equilibrium when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. Equilibrium is not necessarily evolutionarily stable. An *evolutionarily stable stage* is an equilibrium which the dynamic system automatically moves back to if imposed a small deviation. If an equilibrium is not evolutionarily stable, even if the society reaches that point, there is no guarantee that society will stay there forever, because a small turbulence will move the system away and never reaches back. When the population reaches an evolutionarily stable stage, the corresponding strategy played by the population is called *evolutionarily stable strategy* (ESS) (Smith and Price, 1973). By identifying the ESS in a particular interaction between two groups, we can predict the long term outcome of the evolution of society (Chen et al., 2012). The objective of evolutionary game theory is to find the evolutionarily stable equilibrium and its corresponding evolutionarily stable strategy.

If ESS exists in the Manager-Investor game, it has the form of:

$$\begin{aligned} ESS_{percent\ dishonest\ manager} &= \lim_{t\to\infty} x(t) \\ ESS_{percent\ credulous\ investor} &= \lim_{t\to\infty} y(t) \end{aligned} \tag{5}$$

Plugging Table 1, Equation 2 and Equation 3 into Equation 4, we get:

$$\begin{aligned} \frac{dx}{dt} &= x(1-x)((\frac{H-L}{4} + \frac{c}{2})y - \frac{c}{2}) \\ \frac{dy}{dt} &= -y(1-y)((\frac{H-L}{4} - \frac{c}{2})x - \frac{c}{2}) \end{aligned} \tag{6}$$

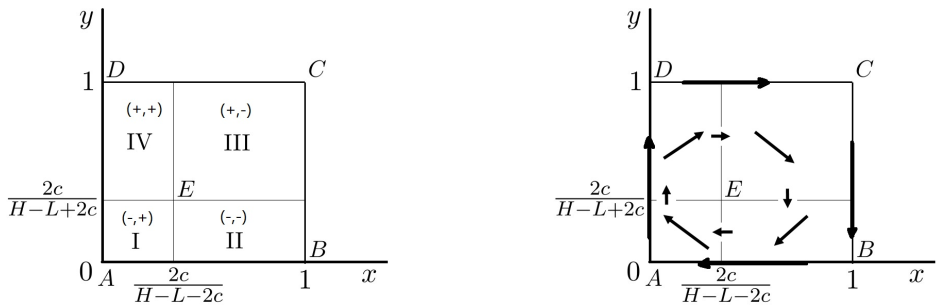
This dynamic system (x, y) has five equilibrium points: $A = (0,0)$, $B = (1,0)$, $C = (1,1)$, $D = (0,1)$ and $E = (\frac{2c}{H-L-2c}, \frac{2c}{H-L+2c})$. These equilibria are not necessarily evolutionarily stable. Note that (x, y) must remain in $\mathcal{P} = [0,1]\times[0,1]$.

THEOREM 3: *If investigation cost is sufficiently small ($c < \frac{H-L}{4}$), neither group in Manager-Investor game has an evolutionarily stable strategy.*

PROOF:

if $c < \frac{H-L}{4}$, then $0 < \frac{2c}{H-L-2c} < 1$. Figure 5 shows the locations of five equilibria. Two lines, $x = \frac{2c}{H-L-2c}$ and $y = \frac{2c}{H-L+2c}$, divide $\mathcal{P}$ into: I, II, III, IV and their boundaries, as shown in Figure 2(a).

We use a pair of signs to express the signs of $(\frac{dx}{dt}, \frac{dy}{dt})$, which indicates the direction of evolution in each section. It can be calculated that section I is $(-,+)$, section II is $(-,-)$, section III is $(+,-)$, and section IV is $(+,+)$. Arrows in Figure 2(a) indicate the direction of social evolution. If the society starts at any of the four corner equilibria, then add a turbulence and move the society a little away from the equilibria point, it will never reach back to that point again. Therefore A, B, C and D are not evolutionary stable strategies. Point E may have Lyapunov



(a) Four subparts of $\mathcal{P}$ (b) A rough graph of the direction field

FIGURE 2. PHASE DIAGRAM METHOD TO SOLVE EQUATION 6 WHEN c IS SMALL

stability. Calculating the Jacobian matrix for point E and we have:

$$J = \begin{pmatrix} 0 & \frac{c(H-L-4c)(H-L+2c)}{2(H-L-2c)^2} \\ \frac{c(L-H)(H-L-2c)}{2(H-L+2c)^2} & 0 \end{pmatrix} \tag{7}$$

It has two imaginary eigenvalues, so no conclusion can be drawn at point E. ∎

REMARK 1: *Calculate the quotient of the two equations in Equation 6, we get:*

$$\frac{dy}{dx} = -\frac{y(1-y)((\frac{H-L}{4} - \frac{c}{2})x - \frac{c}{2})}{x(1-x)((\frac{H-L}{4} + \frac{c}{2})y - \frac{c}{2})} \tag{8}$$

Rearrange Equation 8 we have:

$$\frac{(\frac{H-L}{4} - \frac{c}{2})x - \frac{c}{2}}{x(1-x)}dx + \frac{(\frac{H-L}{4} + \frac{c}{2})x - \frac{c}{2}}{y(1-y)}dy = 0 \tag{9}$$

Notice that if we integrate Equation 9, we will get $h(x, y) = constant$, which is the orbit of $(x(t), y(t))$, where h is a continuous function of x and y. Since there exists a constant term, different values of the constant term define different curves and these curves cannot intersect with each other. Suppose two curves $h(x, y) = constant_1$ and $h(x, y) = constant_2$ ($constant_1 \neq constant_2$) intersect at (x_0, y_0), then $constant_1 = constant_2 = h(x_0, y_0)$ is the conflict. Combined with the direction field shown in Figure 2(b), this indicates the orbit of $(x(t), y(t))$ is a circle surrounding point E within the domain of $\mathcal{P}$.

It shouldn't be surprising that Theorem 3 shows that the society will not evolve into a stable status. Let us jump out of the math and analyze the situation with economic thinking. Assume the society started with a status that most managers are dishonest, and most investors are credulous (III). Since investigation cost is small, investors will soon find out that the expected gain of challenging managers exceeds investigation cost, so investors will begin shifting to incredulous (III to II). With more and more investors challenging the managers, managers will find the expected gain of lying can no longer compensate the loss if being caught lying, so they will shift to be honest (II to I). With fewer and fewer dishonest managers in the society, it becomes less necessary for investors to investigate, so more and more investors become credulous again (I to IV). Finally, managers find out that most investors are easy to con, they will become dishonest (IV to III). The circulation will then start again.

Since the society is evolving on a clockwise circulation in Theorem 3, the deadweight loss, which could be calculated as $E(DWL) = 2c(1-y)(x+0.5(1-x)) = c(1+x)(1-y)$, should also evolve in circulation. To calculate the smallest deadweight loss on the orbit, take the derivative respect to t, we have $\frac{dE(DWL)}{dt} = c(1-y)\frac{dx}{dt} - c(1+x)\frac{dy}{dt}$. One can easily check that $\frac{dE(DWL)}{dt} > 0$ on the up-right part of the orbit and $\frac{dE(DWL)}{dt} < 0$ on the low-left part of the orbit, which means the minimum point should be on the up-left part of the orbit. Therefore, one can improve social efficiency by making the society more honest.

THEOREM 4: *If investigation cost $c \geq \frac{H-L}{4}$, the society will evolve to an evolutionarily stable stage and the corresponding evolutionarily stable strategies are $x = 1$ and $y = 1$.*

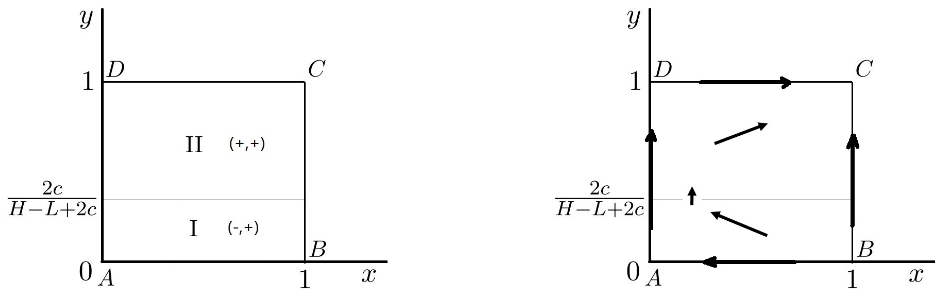
PROOF:

If $\frac{H-L}{4} \leq c < \frac{H-L}{2}$, then $\frac{2c}{H-L-2c} \geq 1$, which means point E is on the right side of $\mathcal{P}$. $x = \frac{2c}{H-L-2c}$ and $y = \frac{2c}{H-L+2c}$ divide $\mathcal{P}$ into section I, section II and their boundaries. The signs of $(\frac{dx}{dt}, \frac{dy}{dt})$ in section I is$(-,+)$ and section II is$(+,+)$. A rough diagram of direction field is shown in Figure 3(b). Arrows in Figure 3(b) indicate the direction of social evolution. The only possible evolutionarily stable stage in Figure 3(b) is point C, which shows as all managers are dishonest and all investors are credulous.

If $c > \frac{H-L}{2}$, then $\frac{2c}{H-L-2c} < 0$, point E is one the left side of $\mathcal{P}$. $\mathcal{P}$ will still be divided into two subparts as shown in Figure 3(a) and it can be checked that the field of line element would still be like Figure 3(b), therefore, C is still the only evolutionarily stable strategy.

If $c = \frac{H-L}{2}$, Equation 6 becomes:

$$\begin{aligned} \frac{dx}{dt} &= x(1-x)(cy - \frac{c}{2}) = 0 \\ \frac{dy}{dt} &= \frac{c}{2}y(1-y) = 0 \end{aligned} \tag{10}$$



(a) Two subparts of $\mathcal{P}$

(b) A rough graph of direction field

FIGURE 3. PHASE DIAGRAM METHOD TO SOLVE EQUATION 6 WHEN c IS LARGE

Solve Equation 10 we can find that point E is no longer an equilibrium and the direction field can still be expressed by Figure 3(b). Therefore, if $c \geq \frac{H-L}{4}$, the society has one and only one evolutionarily stable strategy, which is all managers are dishonest, and all investors are credulous. ∎

In this case, the nature of this game changed. Investors now *agree* that they will only take a fixed return and let the manager keep the remainder of the revenue. A typical example of such situation is commercial banking. People put their money in a banking account and only expect a fixed return even though they understand that the bank is using the deposits to earn a much higher profit. If the investigation cost is too large, people will settle for a "saving account". Another similar scenario is loans. When a firm borrows money from a bank to invest in a project, the bank usually ask for a fixed (series of) payment(s). The amount of money paid back to the bank does not depend how the project goes.

IV. The role of law in the evolve of honesty

From the perspective of policy makers, it is usually considered "good" to have an honest society. This section answers the question: how can policy makers make the society honest and maximize social welfare through laws.

If investigation cost is large, Theorem 4 tells us that investors will naturally agree on a fixed return and let the manager keep everything left, which is a perfectly fine solution to policy makers because both managers and investors are satisfied and no dead weight loss occurred in investigation. If investigation cost is small, society could get to a dishonest stage against investors' will. Ideally, we would hope society remains in the left part of $\mathcal{P}$. But Theorem 3 has shown that even when investigation cost is mall, society will still move to dishonest from time to time. For a policy maker, if the objective is to maximize total social welfare, then there are two approaches: to minimize the investigation cost c or push the society to the up-left corner of $\mathcal{P}$ so investors do not need to investigate. On the other hand, if the objective is to maximize, the investors' welfare, which is often true in real world, the policy maker should make managers less likely to lie and meanwhile reduce the investigation cost for investors. In the following mutant Manager-Investor game, I found that to maximize the welfare of investors, policy makers can pass a law to let managers bear the investigation cost. Suppose a law was passed, requiring the manager cover the investigation cost if being challenged by the investor. The decision tree of this mutant Manger-Investor game is shown in Figure 4. Not much changed from Figure 1, expect that the investors do not suffer any loss when they disbelieve.

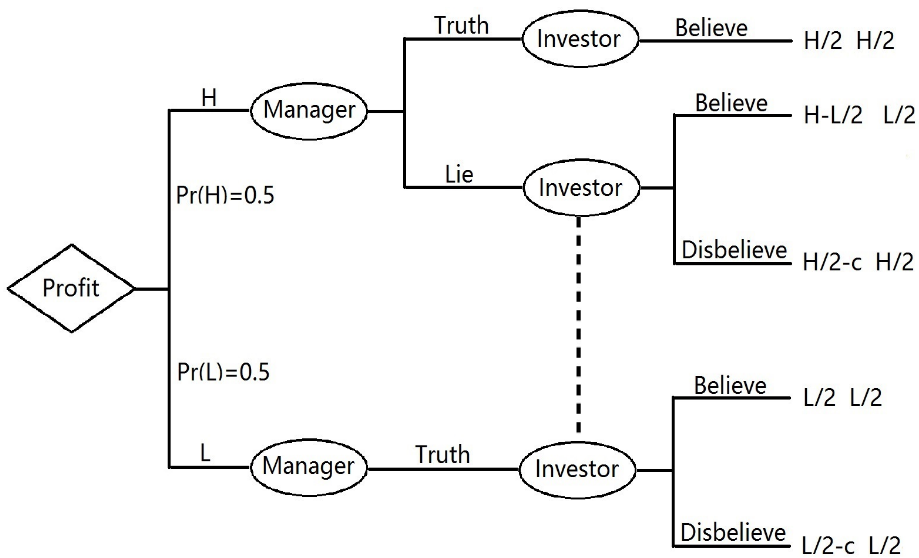


FIGURE 4. TREE OF MANAGER-INVESTOR GAME WHEN MANAGER BEARS ALL INVESTIGATION COST

If society plays new version of Manager-Investor game, the expected payoffs of an M_i meets an I_j will not be the same as in Table 1, the new expected payoffs are calculated in Table 2.

TABLE 2—MATRIX OF EXPECTED PAYOFF WHEN INVESTOR COVERS INVESTIGATION COST

		Receiver	
		I_1	I_2
Investor	M_1	$(\frac{H}{2}, \frac{L}{2})$	$(\frac{H+L}{4} - c, \frac{H+L}{4})$
	M_2	$(\frac{H+L}{4}, \frac{H+L}{4})$	$(\frac{H+L}{4} - \frac{c}{2}, \frac{H+L}{4})$

If Assumption 1 and 2 hold, the investor never believes the manager and the manager never lies. If Assumption 1 is relaxed in the evolutionary approach, investors may not all be incredulous since some investors may find there is no dishonest investor in the society and there is no need to investigate every time. Furthermore, by always being honest, managers can expect investors to gradually build trust and save them the investigation cost. We formalize this intuition in the following theorem.

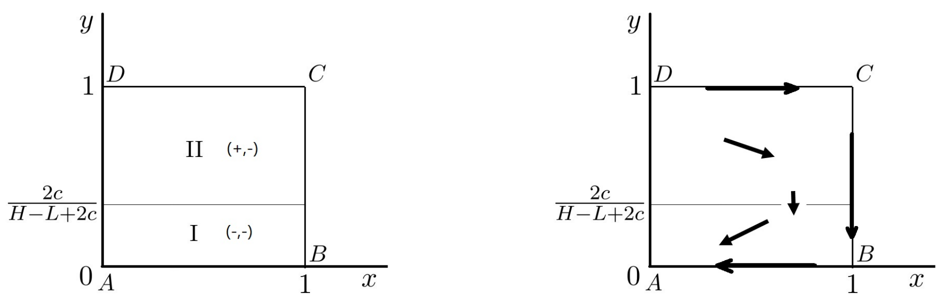
THEOREM 5: *If Assumption 2 holds and managers are responsible for covering the investigation cost for investors, there will be no dishonest manager in the society.*

PROOF:

The replicator dynamics for the new game is:

$$\begin{aligned} \frac{dx}{dt} &= x(1-x)((\frac{H-L}{4}+\frac{c}{2})y-\frac{c}{2}) \\ \frac{dy}{dt} &= -xy(1-y)\frac{H-L}{4} \end{aligned} \tag{11}$$

$\mathcal{P}$ is divided into part I, part II and their boundaries. The signs of $(\frac{dx}{dt}, \frac{dy}{dt})$ is $(+,-)$ in I and $(-,-)$ in II. The direction field is roughly shown in Figure 5(b).



(a) Two subparts of $\mathcal{P}$

(b) A rough graph of direction field

FIGURE 5. PHASE DIAGRAM METHOD TO SOLVE EQUATION 11

Point B, point C and very point on AD satisfy $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. If $(x(t), y(t))$ reaches any of these points, a turbulence will push $(x(t), y(t))$ away and $(x(t), y(t))$ will never reach back, except for point A. Therefore only point A is evolutionary stable. ∎

REMARK 2: *As the society evolve in this new game, it can end up at any point on the vertical axis above* $y = 0$ *and below* $y = \frac{2c}{H-L+2c}$*, which means not all investors are incredulous. This makes sense because investors know that the managers dare not to lie even when they do not investigate, so some investors may* *not investigate. Some managers, also aware of the situation, may take a chance and try lying. In respond to the dishonesty, more investors will immediately s-tart to investigate and push the society back to vertical axis, and the proportion of credulous investors will decrease. If the managers keep taking chances to lie, more and more investors will become incredulous and eventually the society end up at point A. This is why even though every point on AD is equilibrium, but only point A is evolutionarily stable.*

V. Discussion

The Manager-Investor game roughly modeled the evolve of honesty and trust.

I believe that policymakers do not need to regulate the credibility of managers when the investigation cost is large. Managers and their investors will naturally agree on a contract in which the investor only takes a fixed return and let the manager keep the residuals. Just as in saving accounts and loans, a contract of fixed return is provided by the manager and the investor agrees to take it. Default is a different issue in this context, which I believe could also be studied under the framework of Manager-Investor game. The manager could announce default to keep even more profit to himself. But announcing default leads to rigorous investigation in reality and it is profitless for the manager to announce default when he has at least a low profit of L.

If investigation cost is sufficiently small, both groups will try to take advantage of the situation. If managers realize that investors are credulous on average, they will start lying. Investors, on the other hand, will lower their guard if they found managers are mostly honest. This dynamics leads to a circulation shown by Theorem 3. In practice, mandatory auditing in mutual funds has the effect of "killing the circle" and settling for a "good but not best" outcome by forcing the society to stay at point A.

Theorem 5 shows that policy makers can push the society to a honest status by making managers cover the investigation cost for investors. However, as demonstrated by Remark 2, even after the society lands on "all honest" status, it can still experience minor deviations and reverses until finally all investors became incredulous. A policy maker can manually lock the society at point A with mandatory investigation law, which is what SEC does with all the funds in the market. But the SEC does not have authority over private investment agreements like the one between Henry and me in the first section. There are various form of investment agreements that can be described by the Manager-Investor game but are not regulated by the the government. Supervising all kinds of investment agreements could be very costly for the government. Theorem 5 tells us that it is not necessary to supervise every investment agreement in the society. As long as the law requires whoever plays the role as the manager to cover the investigation cost if being challenged, the market will automatically reach to an overall honest stage.

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VI. Appendix